Chapter 5.1

Introduction to Normal Distribution and the Standard Normal Distribution

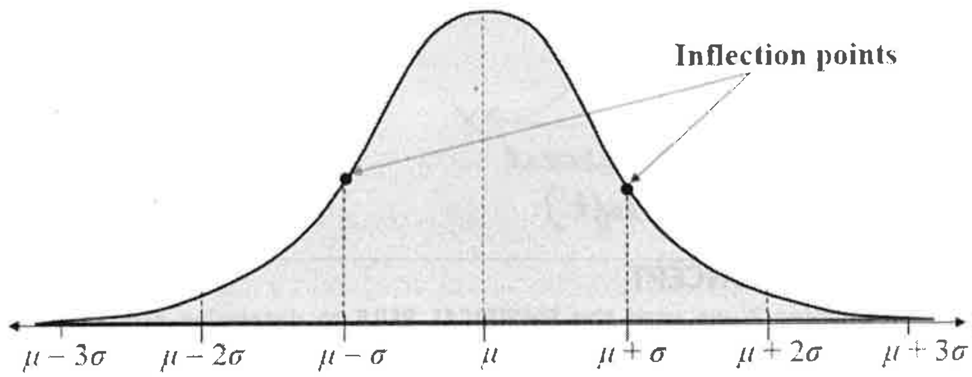
What is a Normal Distribution & a Normal Curve?

-A **normal distribution** is a probability for a **continuous random variable**, *x*.
A **normal curve** is the graph of a **normal distribution**.

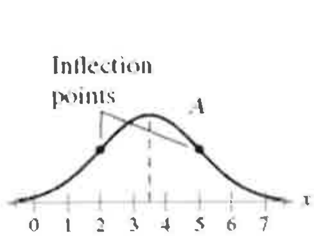
we did discrete variables in Chapter 4.

KEY FEATURES OF NORMAL DISTRIBUTION

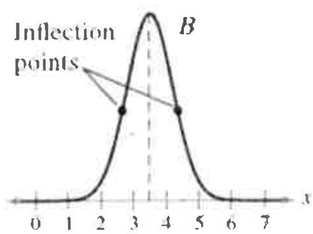
1. Mean, Median, and Mode are equal.
2. The curve is bell-shaped and symmetrical about the mean.
3. The total area under the curve = 1 (or 100%).
4. The curve approaches, but never touches the x-axis.
5. The graph curves downward within one standard deviation of the mean and curves upward outside one standard deviation of the mean. These points are called "inflection points."



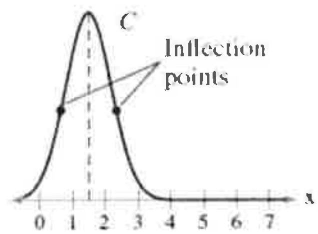
A normal curve can have any mean and any positive standard deviation.



$\mu = 3.5$
$\sigma = 1.5$



$\mu = 3.5$
$\sigma = 0.7$



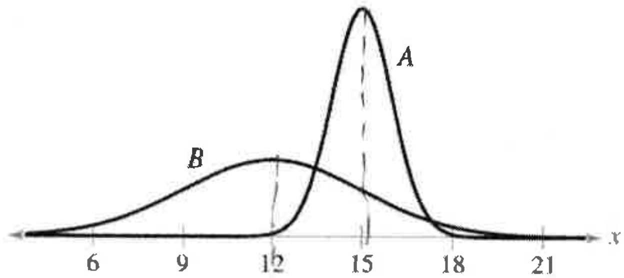
$\mu = 1.5$
$\sigma = 0.7$

Recall the **mean** is a measure of positive: Curves A and B have the same mean.
Recall the **standard deviation** is a measure of spread: Curve A has the largest standard deviation while B and C have the same standard deviation.

Example 1:

Which curve has the greater mean? A

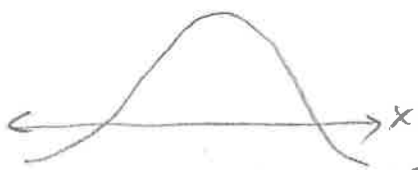
Which curve has the greater standard deviation?
B - it's curve is wider



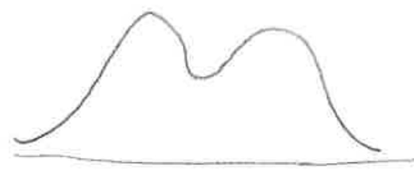
$\mu=12$ (B) $\mu=15$ (A)

Sketches of what a normal curve CAN NOT look like:

not symmetrical (skewed left)



crosses over x-axis



REVIEW CONCEPT

In Chapter 2, we used the **EMPIRICAL RULE** to determine probabilities under normal curves and we reviewed this yesterday in class and for homework.

We also discussed the "**STANDARD SCORE**" OR "**Z-SCORE.**" A z-score tells us "how many standard deviations" a raw score, x, is away from the mean, μ.

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

Example 2: The birth weight of newborns is normally distributed with a mean of 3300 grams and a standard deviation of 600 grams.

a) What is the Z-score for a newborn weight of x = 3900g?

$$z = \frac{3900-3300}{600} = \frac{600}{600} = 1$$

b) What is the Z-score for a newborn weight of x = 2300g?

$$\frac{2300-3300}{600} = \frac{-1000}{600} = -1.67$$

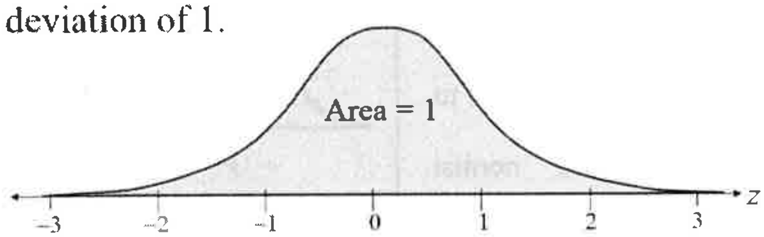
A **"Standard Normal Distribution"** is a normal distribution with a mean of 0 and a standard deviation of 1. The total area under the curve is 1.

If we are dealing with values that have integer Z-scores (-2, -1, 0, 1 etc), we can still use the empirical rule to estimate probabilities.

If we are dealing with ANY Z-score (-2, 0, 1, 1.35, -0.78, etc.) we can use a **"Standard Normal Distribution Table."**

Standard normal distribution

- A normal distribution with a mean of 0 and a standard deviation of 1.

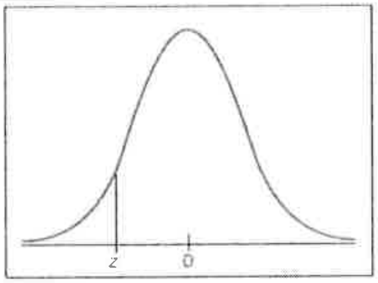


- Any x-value can be transformed into a z-score by using the formula

$$z = \frac{\text{Value - Mean}}{\text{Standard deviation}} = \frac{x-\mu}{\sigma}$$

STANDARD NORMAL DISTRIBUTION TABLE:

Table represents area under the curve & to the left of the z-score.



The table entry for z is the area to the left of z.

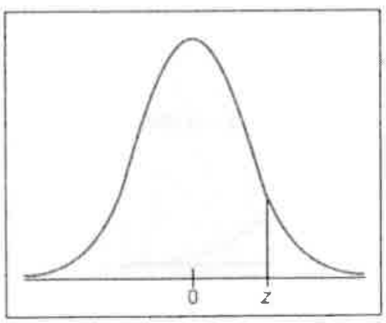
Example: Find the area left of z = -2.94

0.0016

Areas of a Standard Normal Distribution

(a) Table of Areas to the Left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084



The table entry for z is the area to the left of z.

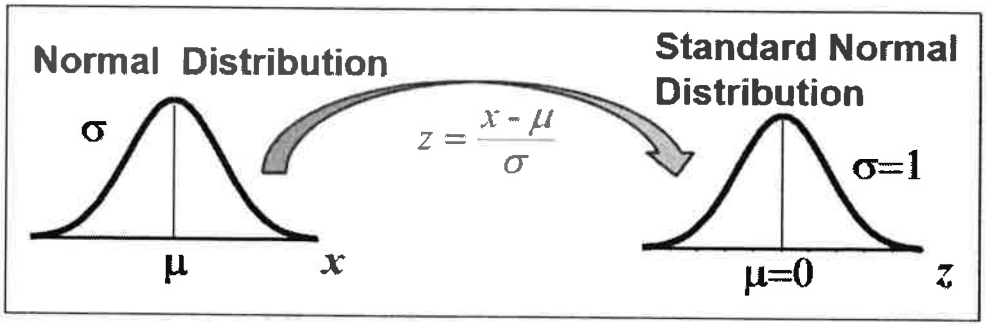
Example: Find the area left of z = 0.37

0.6443

Areas of a Standard Normal Distribution *continued*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

BIG IDEA IN THIS CHAPTER:
GOAL: Estimate probabilities of values of x that are part of a normal distribution.



PROCESS:
*Convert x-values (raw scores) to standard scores (Z-scores).
*Use the standard normal distribution table to look up areas. Areas under the normal curve correspond to probabilities!

HOW TO USE THE TABLE TO FIND PROBABILITIES:

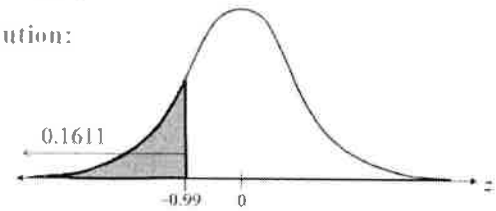
FIRST: Sketch the curve and shade the appropriate areas.

SECOND: Looking for the area on the ...

LEFT SIDE:
Look up the value that corresponds to z in the table. ~~The~~ this is the area to the left of z.

Find the area under the standard normal curve to the left of z = -0.99.

Solution:



From the Standard Normal Table, the area is equal to 0.1611.

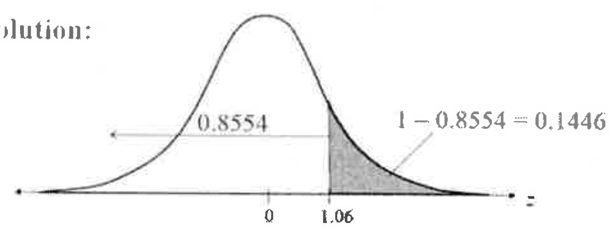
RIGHT SIDE:
Look up the value that corresponds to z in the table. This is the area to the left of z.

Subtract the area to left from 1.

1 – (area to the left) = area to the right.

Find the area under the standard normal curve to the right of z = 1.06.

Solution:



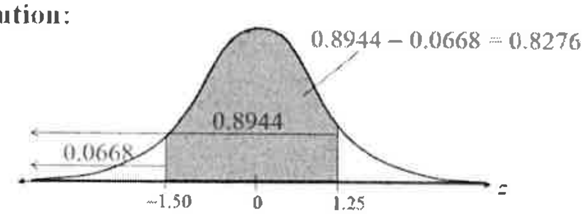
From the Standard Normal Table, the area is equal to 0.1446.

BETWEEN TWO VALUES
Find the area left of both scores. Subtract the smaller area from the larger area.

(larger area) – (smaller area) = (middle area)

Find the area under the standard normal curve between $z = -1.5$ and $z = 1.25$.

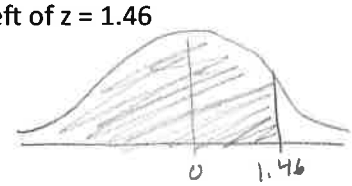
Solution:



From the Standard Normal Table, the area is equal to 0.8276.

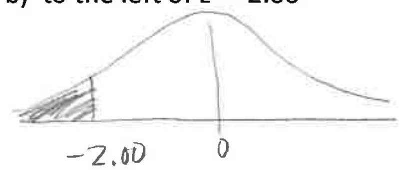
Example 3: Find the area of the indicated region under the standard normal curve. Make a sketch to clearly show the area under the curve that you are finding.

a) to the left of z = 1.46



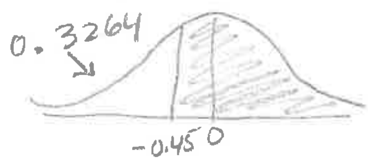
0.9279

b) to the left of z = -2.00



0.0228

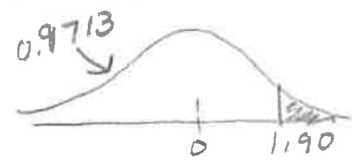
c) to the right of z = -0.45



1 − 0.3264

= 0.6736

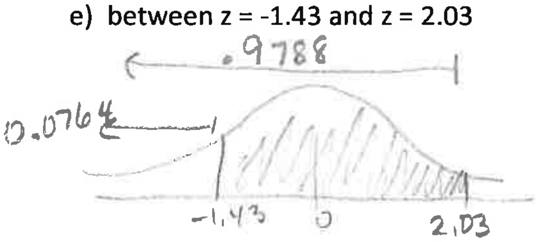
d) to the right of z = 1.90



1 − 0.9713

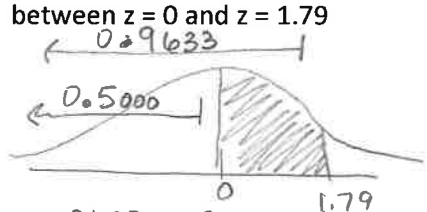
= 0.0287

e) between z = -1.43 and z = 2.03



0.9788 − 0.0764 = 0.9024

f) between z = 0 and z = 1.79



0.9633 − 0.5000 =

0.4633

Because the normal distribution is a continuous probability distribution, the area under the standard normal curve to the left of a z-score gives the probability that z is less than the z-score. For example, the area to the left of z = -0.99 is 0.1611. So, P(z < -0.99) = 0.1611. So from Example 3,

a) $P(z < 1.46) =$ 0.9279

b) $P(z < -2.00) =$ 0.0228

c) $P(z > -0.45) =$ 0.6736

d) $P(z > 1.90) =$ 0.0287

e) $P(-1.43 < z < 2.03) =$ 0.9024

f) $P(0 < z < 1.79) =$ 0.4633